Laws and Theorems of Boolean Algebra

Operations with 0 and 1:

1. \( X + 0 = X \)
2. \( X + 1 = 1 \)

Idempotent laws

3. \( X + X = X \)

Involution law:

4. \( (X')' = X \)

Laws of complementarity:

5. \( X + X' = 1 \)

Commutative laws:

6. \( X + Y = Y + X \)

Associative laws:

7. \( (X + Y) + Z = X + (Y + Z) \)
   \[ = X + Y + Z \]

Distributive laws:

8. \( X( Y + Z ) = XY + XZ \)

Simplification theorems:

9. \( X Y + X Y' = X \)
10. \( X + XY = X \)
11. \( (X + Y') Y = XY \)

DeMorgan’s laws:

12. \( (X + Y + Z + \ldots)' = X'Y'Z'\ldots \)
13. \[ f( X_1, X_2, \ldots X_N, 0, 1, +, \cdot ) \]’ = \( f( X'_1, X'_2, \ldots X'_N, 1, 0, \cdot , +) \)

Duality:

14. \( (X + Y + Z + \ldots)^D = X Y Z \ldots \)
15. \[ f( X_1, X_2, \ldots X_N, 0, 1, +, \cdot ) \]^D = \( f( X_1, X_2, \ldots X_N, 1, 0, \cdot , +) \)

Theorem for multiplying out and factoring:

16. \( (X + Y)(X' + Z) = X Z + X' Y \)

Consensus theorem:

17. \( XY + YZ + X'Z = XY + X'Z \)

18. \( (X + Y)(Y + Z)(X' + Z) = (X + Y)(X' + Z) \)